Mining Spatial Association Rules from Image Databases

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Abstract
In this paper, we propose a mining approach for efficiently finding implicit spatial relations of objects in images. An effective representation for spatial relations is first designed, from which primary spatial relations can be easily obtained. The proposed primary spatial relations have a good characteristic of symmetry, which can greatly reduce the number of candidate itemsets in the mining process. An image mining algorithm is then developed to find the association rules of spatial relations among objects based on the representation. The proposed algorithm first mines object associations and then uses them to find spatial relation associations. The association rules derived may provide some spatial information to appropriate analyzers.

Keywords: data-mining, association rules, spatial relation, spatial mining, image database.

1. Introduction
Recently, data mining has got a great deal of attention as the amounts of data in many applications have grown tremendously large. It can identify effective, coherent, potentially useful, and previously unknown patterns in large databases [2]. Depending on the type of databases processed, the mining approaches may be classified as working on transaction databases, temporal databases, relational databases, and multimedia databases, among others. On the other hand, depending on the classes of knowledge derived, the mining approaches may be classified as finding association rules, classification rules, clustering rules, and sequential patterns [4], among others. Among them, finding association rules in transaction databases is most commonly seen in data mining [1][3][5][9][10][14][15][19][20].

Most previous studies have focused on traditional transaction data. As mentioned above, image data have grown very fast in recent years, so designing an effective mining algorithm from image data may obtain some implicit and useful information in images. It also presents a challenge to workers in the multimedia research field.

In this paper, we thus propose a mining approach for efficiently finding implicit spatial relations of objects in images. A useful representation for spatial relations is first designed, from which primary spatial relations can be easily obtained. The proposed primary spatial relations have a good characteristic of symmetry, which can greatly reduce the number of candidate itemsets in the mining process. An image mining algorithm is then developed to find the association rules of spatial relations among objects based on the representation. The proposed algorithm consists of two main phases. In the first phase, images are regarded as transactions and objects included in the images are regarded as items. The large 1-itemsets and 2-itemsets of objects appearing in the images are then found and act as the input to the second phase. The first phase thus performs a course filtering to ease the later mining process. Then in the second phase, the primary relations for the large 2-itemsets of objects are derived in each
image and are converted into relation items. Large itemsets of relation items are then mined from the images, and association rules of spatial relations can be easily derived from them.

2. Review of Some Related Works

In the past, several famous approaches for effectively processing spatial relations in images were proposed. Chang et al. proposed the 2D-String approach to represent the spatial relations of objects in images [6][7]. Each object in an image is denoted by a symbol. Each image is thus attached two lists of symbols, respectively for the x spatial relations and for the y spatial relations. The symbols in each list are arranged in ascending order according to their position values. Two operators “<” and “=” are used to connect the symbols.

Lee and Hsu then generalized 2D-String and proposed 2D-CString to represent the spatial relations of objects [11][12][13]. A set of spatial operators including “<”, “|”, “=”, “[”, “]”, “%” and “/” is used to identify the spatial relations between objects. Some other variants based on 2D-String were also proposed in the literature [16][18].

Petraglia et al. then proposed the concept of virtual images based on 2D-CString for spatial image retrieval [8]. A virtual image is a virtual description of an actual image. It consists of a set of objects and a set of spatial relations over the objects. The spatial relations are the same as those defined in 2D-CString. Matching between given query images and stored images is performed in their virtual image formats. Similarity measures are also defined among the spatial operators for flexible match.

Rushing et al. proposed a data mining technique to find association rules about adjacent pixels and used them to represent texture features [17]. In their approach, a square window of n by n pixels was taken as a unit of data. The square window moved along the x and y coordinates in an image to get different units of data. These data were then used to get the association rules of the pixel values and their relative positions, representing the regularity of the local structures in the image.

3. Representation of Spatial Relations

In this paper, we propose a mining approach for efficiently finding implicit spatial relations of objects in images. These spatial relations can then be interpreted in a linguistic way, thus much understandable to humans. Some definitions about the representation used in this approach are first given below.

Definition 1: An object is a rectangle segmented out in an image.

Definition 2: A partition $P_{ij}$ of an image $i$ on an object $O_j$ is the $3 \times 3$ regions segmented by the four edges of the objects. Each region is represented by $P_{ij}(x, y), 0 \leq x, y \leq 2$, with the left-top region as $P_{ij}(0, 0)$.

Different partitions are formed on different objects in an image. For example, Figures 1 and 2 show two partitions of an image, respectively on the two objects $A$ and $B$.

![Figure 1: An image partition on object A](image)

![Figure 2: An image partition on object B](image)

Definition 3: In an arbitrary partition $P_{ij}$, the priority of region $P_{ij}(1, 1)$ is 3, the
priority of $P_{ij}(0, 1)$, $P_{ij}(2, 1)$, $P_{ij}(1, 0)$ and $P_{ij}(1, 2)$ is 2, and the priority of $P_{ij}(0, 0)$, $P_{ij}(0, 2)$, $P_{ij}(2, 0)$ and $P_{ij}(2, 2)$ is 1.

Figure 3 clearly shows the priorities of individual regions in a partition. The priorities of individual regions will be used later to define the primary spatial relations between any two objects.

![Figure 3: The priorities of individual regions in a partition.](image-url)

**Definition 4:** The spatial relations of an object $O_j$ on another object $O_k$ in an image $i$ are represented as a set of four-tuples $(O_j, x, y, O_k)$, where $O_k$ overlaps with $P_{ij}(x, y)$.

Take the image in Figures 1 and 2 as an example. The spatial relationship of object $B$ on object $A$ is represented by the set $\{ (A, 2, 0, B), (A, 2, 1, B) \}$ since $B$ overlaps with $P_{1A}(2, 0)$ and $P_{1A}(2, 1)$. Similarly, the spatial relationship of object $A$ on object $B$ is represented by the set $\{ (B, 0, 1, A), (B, 0, 2, A) \}$.

**Definition 5:** The primary spatial relation of an object $O_i$ on another object $O_j$ in an image $i$ is represented as a four-tuple $(O_i, x, y, O_j)$, with region $P_{ij}(x, y)$ having the largest priority among the set of spatial relations of $O_k$ on $O_j$.

Take the image in Figures 1 and 2 as an example. The primary spatial relation of object $B$ on object $A$ is $(A, 2, 1, B)$ and the one of object $A$ on object $B$ is $(B, 0, 1, A)$. These two primary spatial relations can be roughly thought of as the right-of and left-of spatial relations. When objects are represented by rectangles, the primary spatial relation of an object on another one in an image is unique. Using only primary spatial relations in the mining process can thus help avoid redundant patterns and reduce time complexity. It can also be easily inferred from the spatial relations of the edges of the objects that the two primary spatial relations between any two objects are symmetric. We thus have the following definition.

**Definition 6.** Two primary spatial relationships $(O_{ij}, x_i, y_i, O_{kl})$ and $(O_{j2}, x_2, y_2, O_{k2})$ are equivalent if $O_{ij} = O_{k2}$, $O_{kl} = O_{j2}$, $x_1 = 2 - x_2$, and $y_1 = 2 - y_2$.

The two primary spatial relations $(A, 2, 1, B)$ and $(B, 0, 1, A)$ in the image in Figures 1 and 2 are thus equivalent. For a spatial relation $(A, x, y, B)$, one of nine possible combinations of $x$ and $y$ values exists. The first five possible spatial relations are described below.

(a) $(A, 0, 0, B)$: It means that $B$ is to the north-west of $A$ or $A$ is to the south-east of $B$.
(b) $(A, 0, 2, B)$: It means that $B$ is to the south-west of $A$ or $A$ is to the north-east of $B$.
(c) $(A, 1, 2, B)$: It means that $B$ is below $A$ or $A$ is above $B$.
(d) $(A, 0, 1, B)$: It means that $B$ is to the right of $A$ or $A$ is to the left of $B$.
(e) $(A, 1, 1, B)$: It means that $B$ overlaps with $A$ or $A$ overlaps with $B$.

The other four combinations of $x$ and $y$ are symmetric to the first four above. Due to the symmetry of primary spatial relations, the spatial relations between two objects can then be represented by one primary spatial relation, instead of by two. We can replace $(O_i, x, y, O_k)$ with $(O_k, 2-x, 2-y, O_i)$ according to the alphabetic order of $O_i$ and $O_k$. If $O_i$ and $O_k$ represent the same object, we can thus use only the first five spatial relations in the mining process. The following definition for a relation item is thus used.

**Definition 7.** A relation item in the spatial mining process is a spatial relation $(O_i, x, y, O_j)$, with $0 \leq x \leq y \leq 2$, or $(O_j, x, y, O_i)$, where $O_i$ is at front of $O_k$ in the alphabetic order.
Take the image in Figures 1 and 2 as an example. Although the two primary spatial relations \((A, 2, 1, B)\) and \((B, 0, 1, A)\) exist, \((A, 2, 1, B)\) will be used as the relation item in the mining process. \((B, 0, 1, A)\) won’t be used since it is equivalent to \((A, 2, 1, B)\) and \(A\) is at the front of \(B\) by the alphabetic order. Assume in another case, there are two primary spatial relations \((A, 2, 0, A)\) and \((A, 0, 2, A)\). \((A, 0, 2, A)\) will be used as the relation item. In this way, candidate itemsets can be greatly reduced, especially for those with many items.

4. The Proposed Mining Algorithm for Spatial Relations of Objects

In this section, a mining algorithm for spatial relations of objects is proposed to discover useful association rules that describe frequent spatial relations of objects in an image database. The proposed spatial mining algorithm is described as follows.

The proposed spatial mining algorithm:

Input: An image database with \(n\) images, an image table describing the information of objects in the images, a minimum support value \(\alpha\) and a minimum confidence value \(\lambda\).

Output: A set of spatial association rules.

STEP 1: Collect the object names appearing in the images and call them object items.

STEP 2: Consider each image as a transaction of objects consisting of the image identification and the object items included in the image.

STEP 3: Find the count \(t_{ck}\) of each object item \(t_k\) appearing in the set of transactions of objects; set the support \(t_{sk}\) of each item \(t_k\) as \(t_{ck}/n\).

STEP 4: Check whether the support \(t_{sk}\) of each item \(t_k\) is larger than or equal to the predefined minimum support \(\alpha\). If \(t_{sk}\) is equal to or greater than \(\alpha\), put \((t_{m}, x, y, t_{n})\) in the set of large \(2\)-object-itemsets \(OL_2\).

STEP 5: Find the primary spatial relation \((t_{m}, x, y, t_{n})\) of each large \(2\)-object-itemset \((t_{m}, t_{n})\) in each image. Consider each image as a transaction of relations, consisting of the image identification and the primary spatial relations in that image. Call the primary spatial relations relation items.

STEP 6: Find the count \(r_{ck}\) of each relation item \(r_k\) appearing in the set of transactions of relations; set the support \(r_{sk}\) of each relation item \(r_k\) as \(r_{ck}/n\).

STEP 8: Check whether the support of the relation item \((t_{m}, x, y, t_{n})\) is larger than or equal to the predefined minimum support \(\alpha\). If the support of \((t_{m}, x, y, t_{n})\) is larger than or equal to the predefined minimum support \(\alpha\), put \((t_{m}, x, y, t_{n})\) in the set of relation large \(1\)-relation-itemsets \(RL_1\).

STEP 9: Set \(p = 1\), where \(p\) is used to represent the number of items kept in the relation-itemsets currently being processed.

STEP 10: Generate the set of candidate \((p+1)\)-relation-itemsets \(RC_{p+1}\) from \(RL_p\) in a way similar to that in the Apriori algorithm. Find the support of each candidate \((p+1)\)-relation-itemset and check whether the support is equal to or greater than \(\alpha\). Put those with their support values equal to or greater than \(\alpha\) in the set of large \((p+1)\)-relation-itemsets \(RL_{p+1}\).

STEP 11: If \(RL_{p+1}\) is null, then do the next step; otherwise, set \(p = p + 1\) and
repeat STEPs 10 and 11.

**STEP 12:** Construct the spatial association rules for each large \( q \)-relation-itemset \( s \) with relation items \( (s_1, s_2, \ldots, s_q) \), \( q > 1 \), using the following substeps:

(a) Form all possible spatial association rules as follows:
\[
s_1 \Lambda \cdots \Lambda s_{k-1} \Lambda s_{k+1} \Lambda \cdots \Lambda s_q \rightarrow s_k, \quad k = 1 \text{ to } q.
\]

(b) Calculate the confidence values of all spatial association rules by:
\[
\text{count of } (s_1 \Lambda \cdots \Lambda s_{k-1} \Lambda s_{k+1} \Lambda \cdots \Lambda s_q) \quad \text{count of } (s_1 \Lambda \cdots \Lambda s_q).
\]

**STEP 13:** Output the rules with confidence values larger than or equal to the predefined minimum confidence value \( \lambda \).

### 6. An example

Assume an image database includes nine images \( I_1 \) to \( I_9 \), which are shown in Figure 4.

![Figure 4: An image database used in this example.](image)

Assume each object in an image has been segmented out as a rectangle. Also assume that the predefined minimum support value is \( 4/9 \). The minimum count required is then \( 4/9 \times 9 \), which is 4. The minimum confidence value is set at 0.8. The rules with confidence values larger than or equal to the predefined minimum confidence value are output to users. In this example, the resulting spatial association rules are listed in Table 1.

<table>
<thead>
<tr>
<th>The association rules</th>
<th>Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>((A, 0, 1, C) \land (A, 1, 2, H) \Rightarrow (C, 2, 2, H))</td>
<td>4/4</td>
</tr>
<tr>
<td>((A, 0, 1, C) \land (C, 2, 2, H) \Rightarrow (A, 1, 2, H))</td>
<td>4/4</td>
</tr>
<tr>
<td>((C, 2, 2, H) \land (A, 1, 2, H) \Rightarrow (A, 0, 1, C))</td>
<td>4/4</td>
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<td>4/4</td>
</tr>
<tr>
<td>((C, 2, 2, H) \Rightarrow (A, 0, 1, C))</td>
<td>4/4</td>
</tr>
</tbody>
</table>

The first spatial association rule can be interpreted as “If object \( C \) is to the left of object \( A \) and object \( H \) is below \( A \), then object \( H \) is in the south-east of object \( C \)”. The other rules may be explained in a similar way.

### 7. Conclusions and Future Works

In this paper, we have designed a representation to effectively manage spatial relations of objects. We have also proposed a mining algorithm based on this representation to obtain spatial regularity of objects from an image database. The proposed algorithm first mines object associations and then uses them to find spatial relation associations. The association rules derived may provide some spatial information to appropriate analyzers.

### References


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